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Why only few are so successful ?

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Abstract

In many professons employees are rewarded according to their relative performance. Corresponding economy can be modeled by taking N independent agents who gain from the market with a rate which depends on their current gain. We argue that this simple realistic rate generates a scale free distribution even though intrinsic ability of agents are marginally different from each other. As an evidence we provide distribution of scores for two different systems (a) the global stock game where players invest in real stock market and (b) the international cricket.

1 Introduction

In equilibrium systems power law distributions are observed at the criticality. Many open and driven systems in nature, however, naturally self organize to produce scale free distributions[1]. Distribution of rain fall, magnitude of earthquake, link distribution of world wide web are few examples to mention. The scale invariant distributions are also seen in social and economic systems[2]. About a century ago Vilferdo Pareto pointed out that wealth w in any society is distributes as $P(w) = w^{-\gamma}$. Several attempts have been made earlier to understand this phenomena, namely 'the Pareto-law for the wealthy people'[3]. An interesting analogy has been drawn [4] between the economic system and the system of ideal gases where particles and their energies are modelled as agent and their wealth, and redistribution of energy during collision is modelled as trading between agents. This analogy, which naturally generate Gibb's distribution, could successfully explain 97% of the observed income distribution. The rich (about 3%), however, follow a scale-free distribution which was explained later [5] using ideal-gas like models.

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In this article we argue that economy works differently at different levels. In particular for rich and successful it is quite a different game. There are certain kind of occupations, for example the law, the medicine and the journalism, market pays individuals not according to their absolute performance, but according to their performance relative to others in the same occupation. The same is true in the sports, share and entertainment industries. These systems where *"winner plays an important role in the market"* may be named as celebrity markets (CM)[6].

How does a system (or market) generates a successful professional or celebrity ? Of course, there are exceptionally brilliant and strategic individuals who play and controls the market. But often, strategy of players or agents in the market are not very different from each other. However, the distribution of their success or wealth is highly asymmetric with power law tails, where most become unsuccessful and only a few become successful. To understand this phenomena we introduce a simple model of N agents in section 2. In section 3 the theoretical results are compared in certain example systems belongs to this class, namely celebrity markets. Finally the conclusion and some discussions are given in section 4.

2 The model

Let us take an unbiased sample of N agents, labeled by $i = 1, 2, \ldots, N$, who invests equal amount in the market (*say*, stock market). The net gain of the agents$\{m_i\}$ are taken to be integers for simplicity and set $\{m_i = 0\}$ at time $t = 0$. In each time step dt a randomly chosen agent j first decides with probability z, if he wants to continue investing. With probability $1-z$ the agent become inactive forever. If active, the net gain of the agent m_j is increased by unity, with a probability $w(m_j)$. Of course, $\{w(m_j)\}$ are normalized such that total probability of all active agents is unity.

It is reasonable to assume that the growth rate of agents $w(m)$ depends on the the instantaneous gain m and that $w(m)$ is an increasing function. Because, if $m_j(t) > m_i(t)$, agent j can be considered strategically smarter (who studies the market better) than agent i at time t and thus $w(m_j) > w(m_i)$.

Note that, there is no direct interection between agents. The only interaction comes from the fact that the growth rate is relative. Thus our model is an ensemble of N independent agents where wealth m of an agent follows a discrete time dynamics

$$(m-1) \xrightarrow{zw(m)} m, \tag{1}$$

where $w(m)$ is an increasing function. Depending on their asymptotic limits, increasing functions may be classified into two categories; (a) when $w(\infty)$ is ∞ and (b) $w(\infty)$ is finite (say, unity). It is obvious that for case (a), the growth rate $w(m)$ for the smartest agent, chosen stochastically by the process, is large compared to the rest and thus he predictively wins the market. In the second case, where $w(m)$ is a marginally increasing function, probability of gaining an extra unit is comparable among agents, which mimics the cometition existing in real markets. Moreover, we have assumed that the agents are strategically similar and thus choice (b) is more appropriate.

Since agents are independent, $Prob.(\{m_i\}) = \prod_i p(m_i)$, where $p(m)$ is the probability that an agent gains m unit of wealth. To gain m units, one must go through the process $0 \to 1, 1 \to 2, \ldots (m-1) \to m$, which occurs with rate $w(1), w(2) \ldots w(m)$ respectively. Thus the normalized probability is,

$$p(m) = \frac{z^m \prod_{k=1}^m w(k)}{F(z)} \quad \text{where} \quad F(z) = p(0) + \sum_{m=1} z^m \prod_{k=1}^{m} w(k). \tag{2}$$

The average gain $\rho(z) = \langle m \rangle = zF'(z)/F(z)$ is monotonically increases starting from $\rho(0) = 0$. Since maximum value of z is 1 (when agents keep on investing indefinitely), the maximum average gain is $\rho(1)$. If $\rho(1) = \infty$, one can fix any arbitrary density by suitably choosing z. But when $\rho(1)$ is finite, say $\rho(1) = \rho_c$, it is impossible to have uniform macroscopic density $\rho > \rho_c$. Thus in this case, the extra gain $(\rho - \rho_c)N$ would be owned only by one or few agents. In next section we will discuss about such a possibility, namely the condensation of wealth.

2.1 Condensation ?

Let us take $z = 1$. Then agents do not have a choice but to invest indefinitely. Let us further impose an condition that the agents keep on investing until total gain becomes $\sum m_i = M$. Now, the partition function of an ensemble of systems with total gain M being conserved is then

$$Q_M = \prod_{k=1}^{M} w(k). \tag{3}$$

One may consider (3) as a canonical partition function and clearly (2) represents the grand canonical partition function of this system with fugacity z.

In this canonical ensemble one can choose the density $\rho = M/N$ arbitrarily large. When $\rho > \rho_c$, with ρ_c being finite, we have extra wealth $(\rho - \rho_c)N$

which can not be distributed macroscopically. Some agent(s) would gain this macroscopic amount. It can be argued[7] that in the thermodynamic limit, wealth would preferably go to one agent instead being distributed between few agents.

The possibility of having condensation (or a *super celebrity*) depends on the rate $w(k)$. First we need that $\rho_c = \lim_{z\to 1} \frac{zF'(z)}{F(z)}$ is finite. Since $F(z)$ is analytic for $z < 1$, we must check whether $zF'(z) = \sum_1^\infty m z^m Q_m$ is finite as $z \to 1$. This series will converge, when ratio of successive terms decay more slowly than $1+1/m$. Thus asymptotically $w(m)$ should increase faster than $1-2/m$. It is evident from the Taylor's series of $w(m) = 1 - w_1/m - w_2/m^2 \ldots$ that condensation would occur when $w_1 > 2$.

To demonstrate the condensation, let us make a simple choice

$$w(m) = m/(m+b) \tag{4}$$

which is a marginally increasing function. In this case $w_1 = b$ and thus condensation occurs for large density $\rho > \rho_c$, if $b > 2$. To calculate ρ_c, first note that

$$Q_m = \frac{\Gamma(m+1)\Gamma(b+1)}{\Gamma(m+b+1)} \tag{5}$$

Thus, $F(1) = b/(b-1)$ and $F'(1) = F(1)/(b-2)$ and hence, $\rho_c = F'(1)/F(1) = (b-2)^{-1}$.

Any other choice of rate where coefficient of m^{-1} in Taylor's series of $w(m)$ is $-b$ is similar to (4) except that ρ_c is different from $(b-2)^{-1}$. Thus we will continue further discussions with choice (4) .

Distribution of wealth can be obtained from (5). Asymptotically, $Q(m) \propto m^{-b}$. Thus, up to a normalization constant $F(z)^{-1}$,

$$p(m) = z^m Q(m) = z^m m^{-b}. \tag{6}$$

Note that $p(m)$ is similar to observed economic distribution: an exponential distribution for small m and a power-law in the tail.

From (6) one can show that $\langle m \rangle$ diverges for $b > 2$ and thus, in this case condensation occurs for sufficiently large densities. Since, many observed distributions follow Pareto law $p(m) \propto m^{-b}$ with $2 < b < 3$, condenseation of wealth is expected in these economic systems if per capita income (i. e., the density ρ) is very large. Condensation in economic systems has been observed

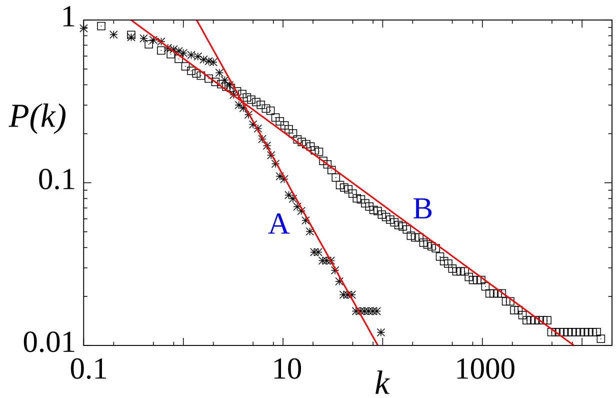

Fig. 1. Probability of gaining more than $k\%$ in global stock game[9]: (A) Group of K-12 students (total373 members, started in January, 2006, starting wealth is \$10,000 per student). (B) Group of 1294 members, started in January 2005, starting wealth is \$666666666 per person.

and modelled earlier[8]. It was argued that the macroscopic accumulation may be viewed as existing corruptions in societies. Here, we show that such a phenomena can occur naturally in CMs.

3 Evidence

As we have discussed earlier, in many occupations, like the medicine the journalism, the share trading and in the entertainment industries like the sports and the film industries, *"winner takes all the market"*. In this section we would cite some examples which are close to the model discussed here.

3.1 Global stock game

Let us take an hypothetical example where brokers invest the same amount of money in stock market. What would be the distribution of their net gain ? It is difficult to carry out such a controlled experiment where (real) money is involved. However a prototype experiment is an existing game, namely global stock game (gsg)[9]. When a group joins this game they get a fixed amount of *gsg dollars* to make transactions in the real stock market (NSE) and individual NAV is evaluated. If these money would have been real the player could have earned the NAV. This game is usually played by (A) a group of school/college students as an learing exercise or (B) by a group of brokers to get experienced in thereal stock market without loosing money. We have collected data for both the groups.

Group A : We have combined two group of K-12 students and calculated the

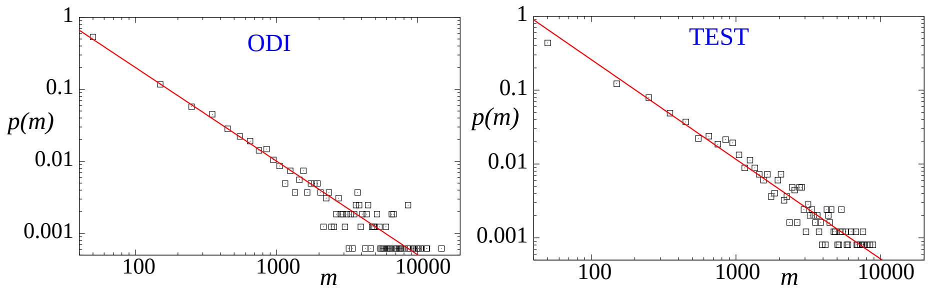

Fig. 2. Distribution of runs: (ODI) The one day international cricket, played by 21 countries having 1620 players; (TEST) The test cricket which is played by 10 countries with 2482 players. Data is collected from *http://www.howstat.com*.

percentage of gain m. If probability of net gain $p(m) \sim m^{-b}$, the probability of gaining more than $k\%$ is $P(k) \sim k^{-b+1}$. Thus, from the plot of $P(k)$ versus k in log scale one can obtain b. Condensation is expected for $b > 2$. In this case, $P(k)$ do not vanish for large k and a correct fitting function is $P(k) = c_1 + c_2 k^{-b+1}$. In fact for group A (see Fig. 1) we find $b = 2.10 \pm .05$, with $c_1 = 0.06$ and $c_2 = 1.35$.

Group B : This a single group of 1294 members joined to win the contest *money666*. For this group we find (see Fig. 1) that $b = 1.43 \pm .05$, with $c_1 = 0$ and $c_2 = 0.52$.

Note the exponent b is different for different groups. A possible reason for group A to have $b > 2$ is that it consists of beginners, where some learners are smarter than others and play well enough to become the super celebrity. However group B consists of exparts (probably) and thus nobody gains exceptionally different from others, which explains the smaller value of b

3.2 International cricket

Another example is the score (e. g., run) of cricketers in international cricket, a bat and ball sport played between two teams of eleven players each. Readers unfamiliar with the game may look at *http://en.wikipedia.org/wiki/Cricket* for details. In this example, let us take two cricketers who started playing about the same time. Obviously one who scores better gets selected for the next international match. Thus, the rate of increase of score $w(k)$ is an increasing function. Again, the difference between rates of two cricketers already having huge runs is small and hence the choice (4)is reasonable.

We have collected life time score (run) of the international cricketers for both one day international (ODI) and the test cricket. The calculated distribution of runs $p(m)$ are plotted in log scale (Fig. 2). Corresponding slopes are found to be $b = 1.3 \pm .05$ for ODI and $b = 1.35 \pm .05$ for test. Since $b < 2$, we do

not expect a super celebrity here. It means, we will never fine an exceptional cricketer (both in ODI and in test cricket) who could score strikingly different from others.

4 Conclusion and Discussion

In this paper we have introduced a model a markets, where agents (employees or players) are rewarded according to their comparative performance. A player who is successful at present has a better probability of being successful in future. We show that this underlying mechanism "success comes easily to people who are already successful" generates a skew distribution even though the ability of players are marginally different from each other. The model successfully describes observation of wealth condensation in economic systems. It also predicts that, in cricket, a brilliant performance which is strikingly different from others, is not expected from any player.

This model is general enough to describe different systems having different b. It would be nice to obtain data at different intermediate times so that from the evolution of the net gain one can calculate $w(k)$ and thus b directly. Such studies would certainly justify the model better. There are many other systems which are test ground for CM model. One example is the distribution of citation of different papers (not authors) where a better cited article is expected to get more citations in future. Another example is the distribution of chromosomal changes per tumor[10] in different kinds of cancers which show a power-law. Here, the mechanism is the following. A cell having more aberrant chromosomes m, would generate daughter cells with more aberrations compared to a cell having smaller m. Thus (4) is a reasonable choice.

It is worth noting that the CM model can be mapped to a well-known model in non-equilibrium studies namely zero range process (ZRP)[11]. ZRP is defined on a one dimensional periodic lattice with N sites and M particles initially distributed randomly among sites. The dynamics of the model is as follows. One particle is transfered from a randomly chosen site to it's rightward neighbour with a rate $u(m)$ where m is the number of particles in the departure site. It can be shown, that the steady state distribution of particles follow (5) in canonical or (2) in grand canonical ensemble with $w(m) = u(m)^{-1}$. The condensation criteria of CM model is identical to that of the ZRP.

The celebrity market model introduced here can also be used to model anomalous diffusion, which will be discussed elsewhere.

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